Double Diffusion in Arbitrary Porous Cavity: Part II

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Abstract. Heat and mass transfer in porous medium is one of the fundamental topics of interest. The present article is dedicated to study the effect of a small block placed at center of left vertical surface of the cavity. The block is maintained at isothermal temperature T_h at three of its edges attached with porous medium. The left surface of cavity is maintained at highest concentration and right surface at lowest concentration. The right surface of cavity is at cold isothermal temperature T_c. Governing equations are converted into matrix form of equations with the help of finite element method and solved iteratively by using a computer code generated in MATLAB.

Keywords. Double diffusion, Irregular square cavity, porous medium

INTRODUCTION

Heat transfer in porous medium has generated tremendous interest among researchers due to its applicability in variety of fields. There are numerous articles that discuss the heat related issues of porous medium [1-36]. The transfer of mass from high concentration region to the region of low concentration is caused by mass diffusion which is also true for the case of porous region. Double diffusion or heat and mass transfer in porous medium is mathematically modeled with the help of an additional equation of mass diffusion [16, 20] along with the momentum and energy equations. The mass transfer is generally expressed either in the form of mass distribution which is represented as iso-concentration lines or Sherwood number at the surface maintained at highest concentration inside the porous region. The heat and mass transfer in porous medium is generally studied for the cases of aiding flow as well as opposing flow which is a result of two buoyancy forces i.e. thermal and concentration either acting in same direction or opposite direction. This article investigates the heat and mass transfer behavior inside the porous medium fixed in a cavity but having a small heating block at the center of left surface. This is an extension to our work to investigate the effect of heat transfer at center of cavity.

MATHEMATICAL MODEL

The geometry of the problem under investigation is shown in figure 1 that shows a small heating block placed at the center of left wall creating an irregular cavity. The various parameters of interest are $\overline{T}, \overline{\psi}, \overline{C}, Ra, N, Rd, Le$ which are non-dimensional temperature, stream function, concentration, Rayleigh number, Buoyancy ratio, radiation parameter and Lewis number. The mathematical equations governing the heat and mass transfer in non-dimensional form can be given as:

$$\frac{\partial^2 \overline{\psi}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\psi}}{\partial \overline{y}^2} = -Ra\left[\frac{\partial \overline{T}}{\partial \overline{x}} + N\frac{\partial \overline{C}}{\partial \overline{x}}\right] \tag{1}$$

$$\frac{\partial \overline{\psi}}{\partial \overline{y}}\frac{\partial \overline{T}}{\partial \overline{x}} - \frac{\partial \overline{\psi}}{\partial \overline{x}}\frac{\partial \overline{T}}{\partial \overline{y}} = \left(\left(1+\frac{4R_d}{3}\right)\frac{\partial^2 \overline{T}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{y}^2}\right) \tag{2}$$

$$\frac{\partial \overline{\psi}}{\partial \overline{y}}\frac{\partial \overline{C}}{\partial \overline{x}} - \frac{\partial \overline{\psi}}{\partial \overline{x}}\frac{\partial \overline{C}}{\partial \overline{y}} = \frac{1}{Le}\left(\frac{\partial^2 \overline{C}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{C}}{\partial \overline{y}^2}\right) \tag{3}$$

With the boundary conditions

$$0 \le \overline{x} \le \overline{L}_s \quad and \quad \overline{y} = 0.4 \; and \; \overline{y} = 0.6 \quad \overline{\psi} = 0 \quad \overline{T} = 1 \quad , \tag{5a}$$

$$0 \le \overline{y} \le \overline{L}_h \quad and \quad \overline{x} = \overline{L}_s \quad \overline{\psi} = 0 \quad \overline{T} = 1 \tag{5b}$$

$$\overline{x} = 0, \quad 0 \le \overline{y} \le 0.4 \quad and \quad 0.6 \le \overline{y} \le 1 \quad \overline{C} = 1 \tag{5c}$$

$$\overline{x} = 1 \qquad \overline{\psi} = 0 \quad \overline{T} = 0 \quad \overline{C} = 0 \tag{5d}$$

$$\overline{y} = 0 \; and \; \overline{y} = 1 \quad \overline{\psi} = 0 \quad \frac{\partial \overline{T}}{\partial \overline{y}} = 0 \quad \frac{\partial \overline{C}}{\partial \overline{y}} = 0 \tag{5e}$$

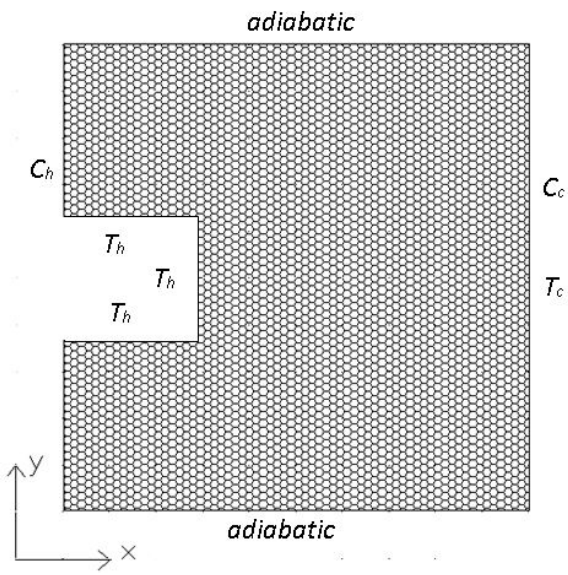

FIGURE 1. Porous domain

RESULTS AND DISCUSSION

The non-dimensional equations of any phenomenon can be easily solved by applying finite element method that results into a set of finite element equations equal to the number of nodes of the whole domain under study. Appropriate boundary conditions as given above are applied to initiate the solution and get the final results in terms of temperature, concentration and stream function distribution at each of the node of physical domain. The heat transfer on top surface of heater significantly reduces due to increase in heater length from 0.2 to 0.5, which is evident from isotherms of Fig. 2 that shows a large region on top of heater being occupied by high temperature lines. Fig. 2 is obtained by keeping the values *Ra=100, Rd=0.1, N=0.2 and Le=2*. The mass transfer has different characteristics on upper and lower side of heater. It is evident that the mass transfer on upper surface increases and lower surfaces decreases with increase in the length of heater as shown by iso-concentration lines of Fig. 2. The fluid velocity at lower side of heater is stronger as compared to upper side as shown by streamlines of Fig. 2. The effect of buoyancy ratio on heat and mass transfer is shown in Fig. 3 which is obtained at *Ra=100, Rd=0.2,* L_s*=23% and Le=2.* The placement of heater at center of left surface leads to large area being occupied with low concentration as compared to the case of heater being placed at the bottom corner of vertical surface. The flow pattern in this case is not much affected due to change in buoyancy ratio. Fig. 4 shows the effect of Lewis number on isotherms, iso-concentration and streamlines for *Ra=100, Rd=1, N=0.1 and* L_s*=23%.* The increase in Lewis number leads to large distortion in concentration lines that in turn increases the mass transfer rate. The central region of cavity possesses very low concentration at higher Lewis number which is quite evident from iso-concentration of Fig. 4.

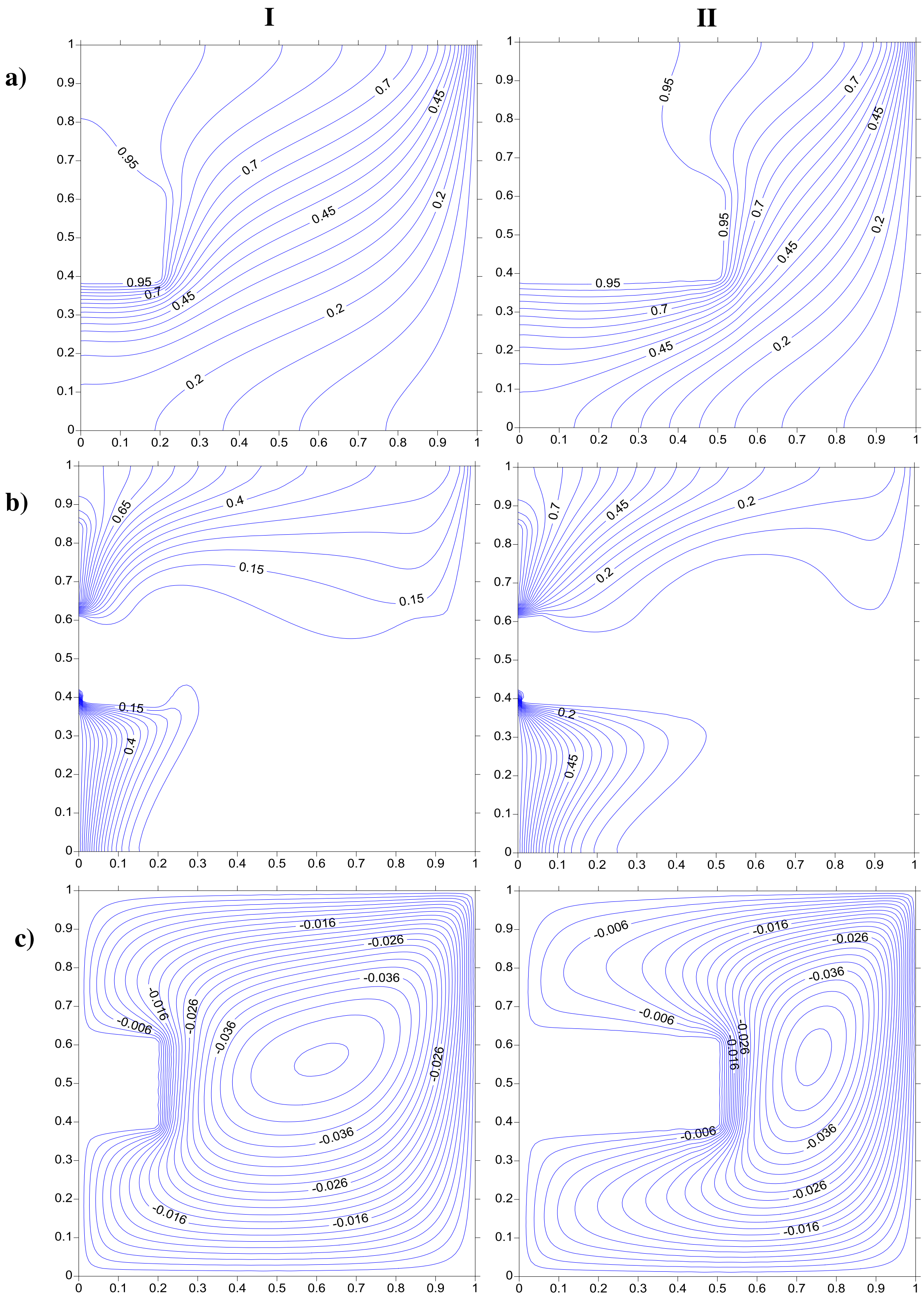

FIGURE 2. Contours a) Isotherms b) Iso-concentration c) Streamlines I) L_s=0.2L II) L_s=0.5L

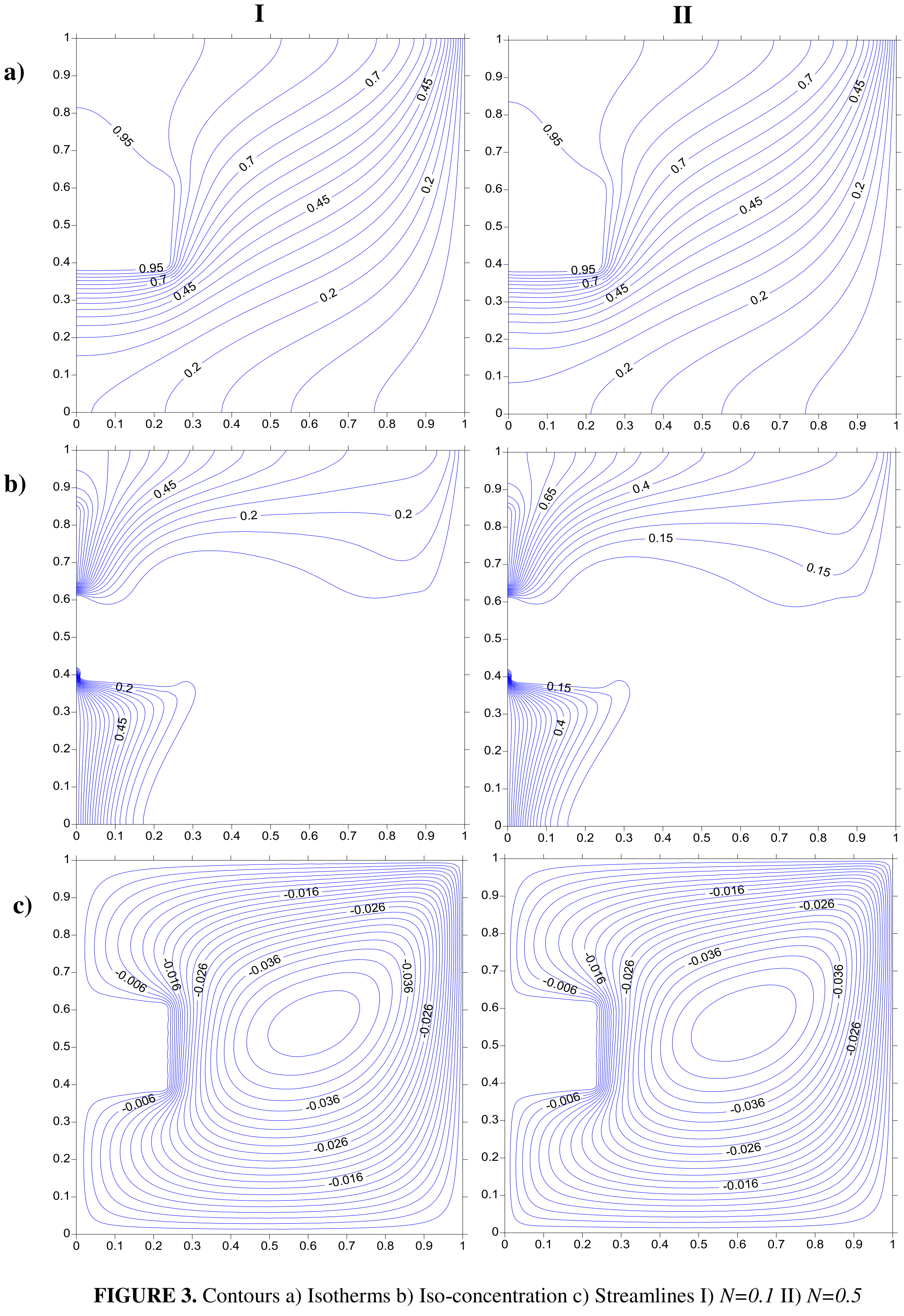

FIGURE 3. Contours a) Isotherms b) Iso-concentration c) Streamlines I) $N=0.1$ II) $N=0.5$

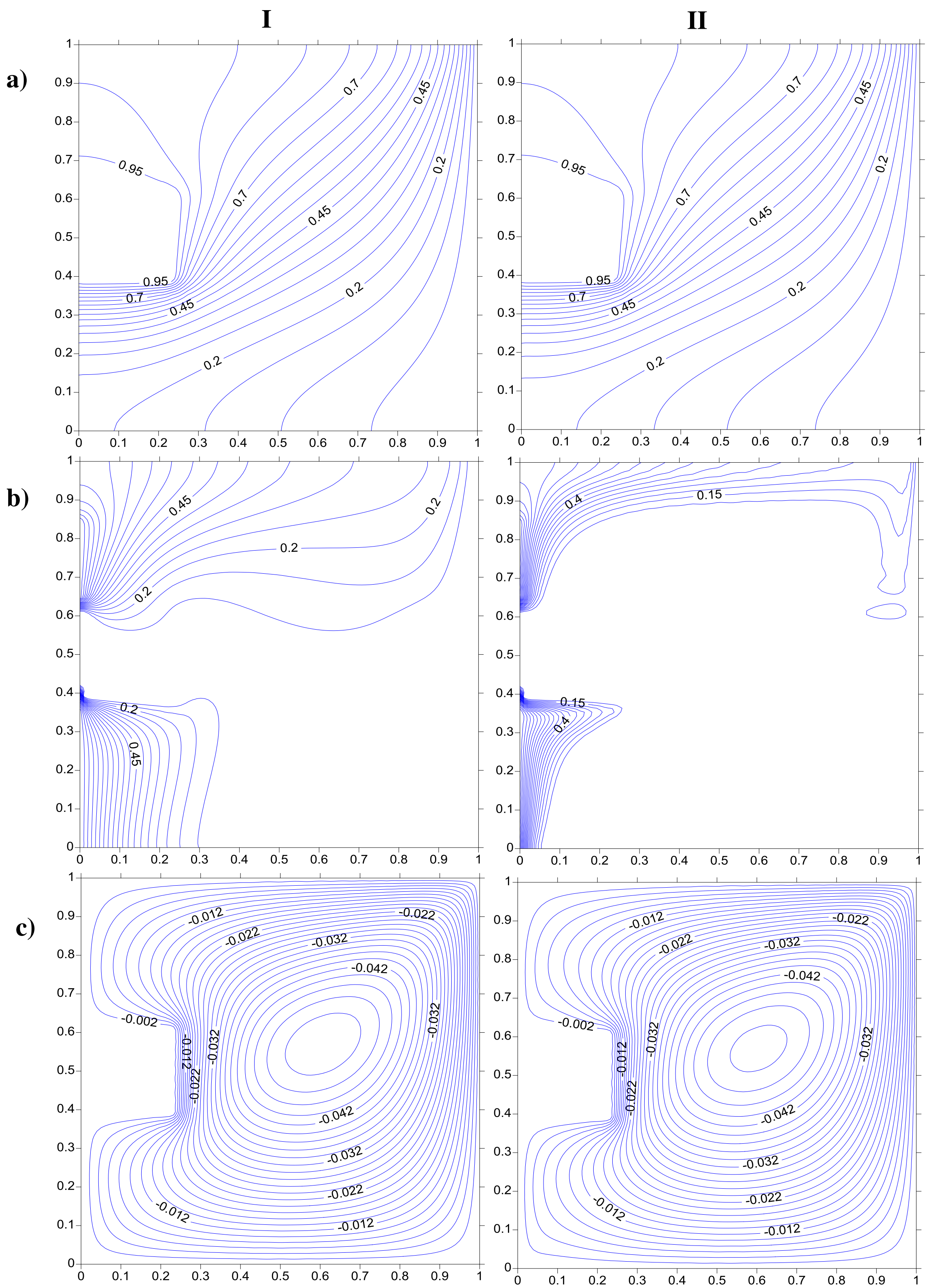

FIGURE 4. Contours a) Isotherms b) Iso-concentration c) Streamlines I) *Le=1* II) *Le=10*

CONCLUSION

Placement of a block shaped heater at center of left surface of porous cavity is investigated with respect to the size of the heater, buoyancy ratio and the Lewis number. It is found that the heat transfer at top surface of heater reduced due to increase in heater length whereas the mass transfer increased with increase in Lewis number.

REFERENCES

1. A.D. Bejan, Kraus (Eds), Heat transfer handbook Wiley, New York, 2003.
2. D.B. Ingham, I.Pop (Eds.),Transport phenomena in porous media, Pergamon, Oxford, 1998.
3. D. Nield and A. Bejan, Convection in Porous Media, ed. 3rd. New York: Springer Verlag.,2006.
4. K .Vafai, Hand book of porous media, Marcel Dekker, New York, 2000.
5. I. Pop, D.B. Ingham, Convective heat transfer: Mathematical and computational modeling of viscous fluids and porous media, Pergamon, Oxford, 2001.
6. Joshi, Y., Nakayama, W. Forced convection: external flows. In: Bejan, A., Kraus, A.D. (Eds.), Heat Transfer Handbook. John Wiley & Sons, Hoboken, p. 443. 2003.
7. Quadir, G. A. and Irfan Anjum Badruddin, Heat transfer in porous medium embedded with vertical plate: Non-equilibrium approach - Part B, AIP Conf. Proc. 1738, 480125 (2016).
8. Badruddin, I.A., Zainal, Z.A., Narayana, P.A., Seetharamu, K.N, Heat transfer in porous cavity under the influence of radiation and viscous dissipation, Int. commun. Heat Mass Transfer. 33(4), 491-499(2006).
9. Badruddin, I.A., Zainal, Z.A., Narayana, P.A., Seetharamu, K.N. and Siew, L.W. Free convection and radiation for a vertical wall with varying temperature embedded in a porous medium. Int. J. Therm., Sci. 45(5), 487-493 (2006a).
10. Badruddin, I.A., Zainal, Z.A., Narayana, P.A., Seetharamu, K.N. and Siew, L.W. Free convection and radiation characteristics for a vertical plate embedded in a porous medium. Int. J. Numerical Methods in Engineering, 65(13), 2265–2278 (2006b).
11. Yunus Khan, T.M., Irfan Anjum Badruddin , Quadir, G.A., Heat transfer in a conical porous cylinder with partial heating, IOP Conf. Series: Materials Science and Engineering 149 ,012211 (2016)
12. Badruddin I.A., Zainal, Z.A., Khan Z. A and Mallick, Z., Effect of viscous dissipation and radiation on natural convection in a porous medium embedded within vertical annulus, Int. J. Therm. Sci. 46 (3), 221-227(2007).
13. Badruddin, I.A., Zainal, Z. A., Narayana, P.A. and Seetharamu, K.N., Thermal non-equilibrium modeling of heat transfer through vertical annulus embedded with porous medium, Int. J. Heat Mass Transfer. 49(25-26), 4955-4965(2006).
14. Ahmed, N.J.S., Badruddin, I.A., Zainal, Z.A., Khaleed, H.M.T. and Kanesan, J. Heat transfer in a conical cylinder with porous medium. International Journal of Heat and Mass Transfer, 52(13-14), 3070-3078 (2009).
15. Ahmed, N.J.S., Badruddin, I.A., Kanesan, J., Zainal, Z.A. and Ahamed, K.S.N. Study of mixed convection in an annular vertical cylinder filled with saturated porous medium, using thermal non-equilibrium model. International Journal of Heat and Mass Transfer, 54(17-18), 3822-3825 (2011).
16. Badruddin, I.A., Ahmed, N. J. S., Al-Rashed, A.A.A.A., Kanesan, J., Kamangar, S. and Khaleed, H.M.T. Analysis of Heat and Mass Transfer in a Vertical Annular Porous Cylinder Using FEM. Transport in Porous Media, 91(2), 697-715 (2012).
17. Al-Rashed, A.A.A.A., Badruddin, I.A. Heat transfer in a porous cavity, International Journal of Mechanical and Materials Engineering, 7 (1), pp. 1-8
18. Badruddin, I.A., Yunus Khan, T. M. , Salman Ahmed N. J and Sarfaraz Kamangar, Effect of variable heating on double diffusive flow in a square porous cavity, AIP Conf. Proc. 1728, 020689 (2016).
19. Irfan Anjum Badruddin and Quadir, G. A., Radiation and viscous dissipation effect on square porous annulus, AIP Conf. Proc. 1738, 480127 (2016).
20. Irfan Anjum Badruddin and Quadir, G. A., Heat and mass transfer in porous cavity: Assisting flow, AIP Conf. Proc. 1738, 480126 (2016)
21. Salman Ahmed N J , Abdullah A AA Al-Rashed , Yunus Khan, T.M, Sarfaraz Kamangar , AbdulgaphurAthani , Irfan Anjum Badruddin, Heat transfer analysis in an annular cone subjected to power law variations, IOP Conf. Series: Materials Science and Engineering 149 ,012212 (2016)

22. Badruddin, I.A., Zainal, Z.A., Narayana, P.A., and Seetharamu, K. N., Numerical analysis of convection conduction and radiation using a non-equilibrium model in a square porous cavity, Int. J. Therm. Sci. 46(1), 20-29 (2007).
23. Badruddin, I.A., Zainal, Z.A., Narayana, P.A., Seetharamu, K.N., Heat transfer by radiation and natural convection through a vertical annulus embedded in porous medium, Int. Commun. Heat Mass Transfer. 33(4), 500-507(2006).
24. Irfan Anjum Badruddin and G. A. Quadir, Heat transfer in porous medium embedded with vertical plate: Non-equilibrium approach - Part A, AIP Conf. Proc. 1738, 480124 (2016).
25. Nik-Ghazali.N., Badruddin.I.A.,Badarudin.A., and Tabatabaeikia.S. Advances Dufour and Soret Effects on Square Porous Annulus. Advances in Mechanical Engineering, January-December. 6 , 209753 (2014).
26. Azeem, T. M. Yunus Khan, I.A. Badruddin, N. Nik-Ghazali and Mohd Yamani Idna Idris, Influence of radiation on double conjugate diffusion in a porous cavity, AIP Conf. Proc. 1728, 020283 (2016).
27. Azeem, Irfan Anjum Badruddin, Mohd Yamani Idna Idris, Nik-Ghazali, N, Salman Ahmed N.J, Abdullah A AA Al-Rashed. Conjugate heat and mass transfer in square porous cavity, Indian Journal of Pure & Applied Physics Vol. 54, pp.777-786 (2016).
28. Ahmed, N.J.S., Kamangar, S., Badruddin, I.A. , Al-Rashed, A. A. A. A., Quadir, G.A., Khaleed, H.M.T. and Khan, T.M.Y., Conjugate heat transfer in porous annulus, J. Porous Media. 19(12), 1109-1119(2014).
29. Badruddin, I. A., Ahmed N. J. S, Al-Rashed, A. A. A., Nik-Ghazali, N., Jameel, M., Kamangar, S., Khaleed, H. M. T., and Yunus Khan, T .M., Conjugate Heat Transfer in an Annulus with Porous Medium Fixed Between Solids, Transport in Porous media, 109(3), 589-608 (2015).
30. Badruddin, I.A., Al-Rashed, A.A.A.A., Ahmed, N.J.S., Kamangar, S. and Jeevan, K. Natural convection in a square porous annulus. International Journal of Heat and Mass Transfer, 55(23–24), 7175-7187 (2012).
31. Badruddin, I.A., Abdullah, A.A.A.A., Ahmed, N. J. S. and Kamangar, S. Investigation of heat transfer in square porous-annulus. International Journal of Heat and Mass Transfer, 55 (7-8), 2184-2192 (2012).
32. Ahmed, N.J.S., Zainal, Z.A., Badruddin, I.A., Hussain, M.T.K. Heat transfer analysis of porous medium in a conical cylinder with variable wall temperature. International Journal of Mechanical and Materials Engineering, 3 (2), 145-152 (2008).
33. Badruddin, I.A., Azeem, Mohd Yamani Idna Idris, Nik-Ghazali. N., Salman Ahmed N.J, Abdullah A.A.A. Al-Rashed, Simplified Finite Element Algorithm to Solve Conjugate Heat and Mass Transfer in Porous Medium, International Journal of Numerical Methods for Heat and Fluid Flow, Vol. 27, (2017).
34. Badruddin, I.A., Zainal, Z.A., Aswatha Narayana, P.A., Seetharamu, K.N, Natural convection through an annular vertical cylindrical porous medium , International Journal of Heat Exchangers .7 (2), pp. 251-262 (2006).
35. LARSON, SE; POULIKAKOS, D, DOUBLE DIFFUSION FROM A HORIZONTAL LINE SOURCE IN AN INFINITE POROUS-MEDIUM, INTERNATIONAL JOURNAL OF HEAT AND MASS TRANSFER Volume: 29 Issue: 3 Pages: 492-495 (1986)
36. SELIMOS, B; POULIKAKOS, D, ON DOUBLE DIFFUSION IN A BRINKMAN HEAT GENERATING POROUS LAYER, INTERNATIONAL COMMUNICATIONS IN HEAT AND MASS TRANSFER Volume: 12 Issue: 2 Pages: 149-158 (1985)