Dynamic Thermal Wave Response and Propagation through Building Structures Using Infinite Elements in Time and Frequency Domain

Konstantin S. Kazakov[1,a)] and Iliana Y. Stoynova[2, b)]

[1]*Professor, D.Sc., Ph.D., C.Eng, VSU"Luben Karavelov", Structural mechanics department, 175'Suhodolska str. 1373 Sofia, Bulgaria*
[a)]kazakov@vsu.bg

[2]*Assist. Professor, C.Eng., VSU"LubenKaravelov", Structural mechanics department, 175'Suhodolska str. 1373 Sofia, Bulgaria*
[b)]*stoynova@vsu.bg*

Abstract. This paper is devoted to a new approach the dynamic termal response and the factor of termal wave propagation through of complex building structure to be evaluated using infinite elements. The far field of such structures is discretized by decay or mapped infinite elements. These elements are appropriate for complex building structures, subjected to termal wave propagation and solved in time or frequency domain. Such infinite elements can be treated as new modified forms of the recently proposed by the first author infinite elements with united shape functions. In the research the time domain form of the equations is demonstrated and used in the numerical example. Only 2D horizontal type infinite elements is used, but by similar techniques 2D vertical and 2D corner infinite elements can also be formulated. The application of the proposed elements in the Finite element method is demonstrated in brief.

Key words: Complex building structures, Wave propagation factor, Infinite elements, The Finite element method, Effective energy solutions

INTRODUCTION AND INFINITE ELEMENT BECKGROUND

The conclusions of the European Council of 2007 provoked the need to increase energy efficiency in the Union to achieve by 2020, 20% of primary energy consumption in the Union compared to projections. Moreover, buildings are crucial to achieving the Union'starget to redinucegasemissionsby 80-95% by 2050 compared to 1990. The implementation of cost-effective energy solutions and the development of measures to improve energy efficiency the buildings are justified by the benefits and savings associated with their implementation [8]. In developing measures to improve energy efficiency should take into account the benefits and savings associated with efficiencies that are achieved through the widespread application of cost-effective technological innovations, [9]. Motivation in the research is based on the idea using recently proposed infinite elements [2,3], to simulate the dynamic thermal response of building structure and predict their efficiency, [10].

Exterior domain problems appear naturally in many engineering fields such as electrodynamics, magnetic problems, fluid flow, thermal analyses, etc. Wave propagation in an elastic infinite media and the scattering of waves on bodies in a fluid which extends infinitely are of particular interest. The main difficulty in such problems when we use numerical methods arises in an unbounded domain that has to be mashed, [7]. Many suggestions for the treatment of the exterior domain have been presented in a number of research papers for the last decades,[5].

The basic idea in this research is to couple the advantages of the Finite element method with some infinite element approximation techniques in order to simulate wave propagation more properly. This simulation is very important in a broad class of problems in computational wave mechanics. The accuracy of the numerical solution depends only on the FEM,[11,12]. From the algorithmic point of view, the infinite element is treated as a standard finite element except for the infinite approximation in the infinite direction.

Infinite elements were originally introduced by Ungeless and Bettess[1]. Now this technique is one of the most often used since its concept, and the formulation is much closer to those of the FEM, [4,6]. These elements are very effective for the modeling of structures with a near field presented by finite elements and a far field presented by infinite elements. The local coordinate system of the proposed infinite element is shown in Fig.1. The geometric mapping from the local coordinates to the global coordinates of infinite element is defined as:

$$x = x_b(1+\xi) \qquad \text{and} \qquad z = \sum_{i=1}^{n} L_i(\eta) z_i \,, \tag{1}$$

where x_b is the global coordinate of the artificial boundary between the near field and the far field, *n* is the number of nodes for the infinite elements, and $L_i(\eta)$ is a Lagrange polynomial, [2]

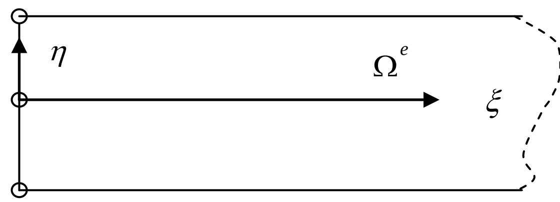


FIGURE 1. Local coordinate system of infinite elements

The geometric transformation along the infinite directions can be realized through two, three or four nodes [3]. Here, in details, only the transformation concerning the first type element is shown. For an element, the domain of which extends to infinity in only one direction (the first and second types), a Lagrange polynomial interpolation in the other direction is used. Here, serendipitous mapping functions are used, written in two different forms

Variant I

$$\begin{gathered} M_1 = -\frac{2\xi}{1-\xi}, \\ M_2 = (1+\xi)\left(\frac{1}{3}-\xi\right)\left(\frac{1}{3}+\xi\right), \\ M_3 = \frac{3}{4}(1+\xi), \\ M_4 = (1+\xi)\left(\frac{1}{3}-\xi\right)\left(\frac{1}{3}+\xi\right), \end{gathered} \tag{2}$$

or

Variant II

$$\begin{gathered} M_1 = -\frac{2\xi}{1-\xi} + \frac{9}{8}(1+\xi)\left(\frac{1}{3}+\xi\right), \\ M_2 = (1+\xi)\left(\frac{1}{3}-\xi\right)\left(\frac{1}{3}+\xi\right), \\ M_3 = \frac{3}{4}(1+\xi), \\ M_4 = (1+\xi)\left(\frac{1}{3}-\xi\right)\left(\frac{1}{3}+\xi\right), \end{gathered} \tag{3}$$

where the difference concerns only the mapping of point x_J .The global mapping [2], using **Variant I** and **Variant II** can be written respectively as

$$x = -\frac{2\xi}{1-\xi}x_J + (1+\xi)\left(\frac{1}{3}-\xi\right)\left(\frac{1}{3}+\xi\right)x_K + \frac{3}{4}(1+\xi)x_L + (1+\xi)\left(\frac{1}{3}-\xi\right)\left(\frac{1}{3}+\xi\right)x_M \tag{4}$$

or

$$x = \left[-\frac{2\xi}{1-\xi} + \frac{9}{8}(1+\xi)\left(\frac{1}{3}+\xi\right)\right]x_J + (1+\xi)\left(\frac{1}{3}-\xi\right)\left(\frac{1}{3}+\xi\right)x_K + \frac{3}{4}(1+\xi)x_L + (1+\xi)\left(\frac{1}{3}-\xi\right)\left(\frac{1}{3}+\xi\right)x_M \tag{5}$$

Mapping functions ensure only the geometric transformation of the coordinates. In mapping only one direction, Jacobian matrices related to two-dimensional and three-dimensional infinite elements are written as

$$[J]^{-1} = \sum_{i=1}^{n}\begin{Bmatrix} x_i \dfrac{\partial M_i}{\partial x}\partial\xi & x_i \dfrac{\partial M_i}{\partial x}\partial\eta \\ \dfrac{\partial\xi}{\partial y} & \dfrac{\partial\eta}{\partial y} \end{Bmatrix} \tag{6}$$

or

$$[J]^{-1} = \sum_{i=1}^{m}\begin{Bmatrix} x_i \dfrac{\partial M_i}{\partial x}\partial\xi & x_i \dfrac{\partial M_i}{\partial x}\partial\eta & x_i \dfrac{\partial M_i}{\partial x}\partial\zeta \\ \dfrac{\partial\xi}{\partial y} & \dfrac{\partial\eta}{\partial y} & \dfrac{\partial\zeta}{\partial y} \\ \dfrac{\partial\xi}{\partial z} & \dfrac{\partial\eta}{\partial z} & \dfrac{\partial\zeta}{\partial z} \end{Bmatrix}, \tag{7}$$

where*n* and *m* indicate the number of functions used in two- and three-dimensional cases respectively, [2].

CONSEPT OF INFINITE ELEMENTS WITH UNITED SHAPE FUNCTIONS

Only some details of the formulation of a horizontal infinite element are demonstrated. Using similar steps the vertical and corner elements can also be formulated. In the beginning, for the sake of convenience, the proposed element is given as decay type. Later the formulation is developed in a mapped form by mapping functions. At the beginning, the transformation, given as

$$x = x_b\left(1+\bar{\xi}\right) \qquad \text{and} \qquad z = \sum_{i=1}^{n} L_i(\eta) z_i \tag{8}$$

is used, where x_b is the global coordinate in the x direction of the artificial boundary between the near field and the far field; *n* is the number of nodes for the infinite elements, and $L_i(\eta)$ is a Lagrange polynomial which has a unit value at the *i*-th node while, zeros are at the other nodes. The wave field in the infinite element [5] can be described in the standard form of the shape functions based on wave propagation functions as

$$\mathbf{u}(x,z,\omega) = \sum_{i=1}^{n}\sum_{q=1}^{m} N_{iq}(x,z,\omega)\mathbf{p}_{iq}(\omega) \text{ or } \mathbf{u}(x,z,\omega) = N_p(x,z,\omega)\mathbf{p}(\omega), \tag{9}$$

where $N_{iq}(x,z,\omega)$ are the standard shape functions; $\mathbf{p}_{iq}(\omega)$ are the generalized coordinates associated with $N_{iq}(x,z,\omega)$; *n* is the number of nodes for the element; and *m* is the number of wave functions included in the formulation of the infinite element. For horizontal wave propagation the basic shape functions can be expressed as:

$$N_{iq}(x,z,\omega)=T(x,z,\bar{\xi},\eta)L_i(\eta)W_q(\bar{\xi},\omega), \quad (10)$$

where $W_q(\bar{\xi},\omega)$ are horizontal wave functions based on the theory of wave propagation in elastic isotropic media with an infinite size of one direction, and $T(x,z,\bar{\xi},\eta)$ is mapping function. For the sake of convenience, the wave functions in the far field region are chosen in an exponential form [2] as:

$$W_q(\bar{\xi},\omega)=e^{-C_s(\omega)\bar{\xi}} \text{ or } W_q(\bar{\xi},\omega)=e^{-C_p(\omega)\bar{\xi}}, \quad (11)$$

where C_s or C_p are complex constants, [5] depending on the wave velocities, the frequencies, and real positive constants which are relative to the geometric attenuation and other parameters of the wave propagation. The real part of the first wave function is shown in *fig.2*.

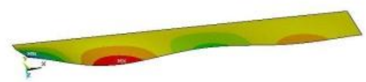


FIGURE 2. The real and imaginary parts of used wave function.

Using only real parts of the wave functions and applying the Euler transformation, the equations of the wave propagation can be written as

$$\operatorname{Re}W(\bar{\xi})=\sum_{q=1}^{m}A_q\cos\left(\frac{\omega_q}{c_s}\bar{\xi}\right)e^{-\alpha\bar{\xi}}$$

or

$$\operatorname{Re}W(\bar{\xi})=\sum_{q=1}^{m}A_q\cos\left(\frac{\omega_q}{c_p}\bar{\xi}\right)e^{-\alpha\bar{\xi}}, \quad (12)$$

where coefficients A_q can be treated as standard Fourier coefficients [2]. Now the sum of eq. (10) for $q=1,2,3,...,m$ can be expressed as

$$N_i(x,z)=\sum_{q=1}^{m}N_{iq}(x,z,\omega)=T(x,z,\bar{\xi},\eta)L_i(\eta)\operatorname{Re}W(\bar{\xi}) \cdot \quad (13)$$

SOME ASSPECTS ON APPLICATION OF PROPOSED INFINITE ELEMENTS IN SIMULATIONS OF ENERGY RENOVATION

Concerned business building was built in 1996 and situated in city of Sofia. It was designed and constructed according current then regulations and requirements for the technical efficiency. Then current requirements for the technical efficiency are quite low in comparison with the requirements placed on energy efficiency at present. The building has four floors and a basement. The built area is 221 sq.m and total area of 1603 sq.m. The construction is realized as reinforced concrete structure with brick external walls are 25 cm in thick.

FIGURE 3. E-N Façade before re-habilitation

The building was renovated in stages, according to currant regulations. Renovation of the building was completed in early Dec. 2010. Filled external insulation of 8 cm. EPS (expanded polystirol) in two stages and mineral plaster was applied, shown in details in Fig.4.

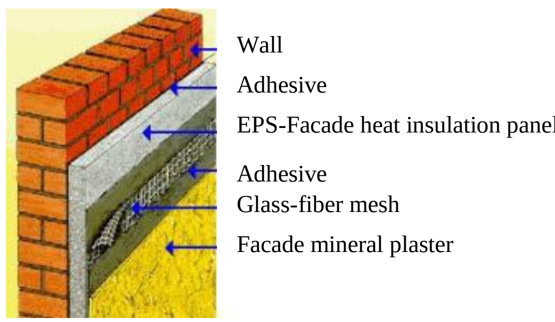
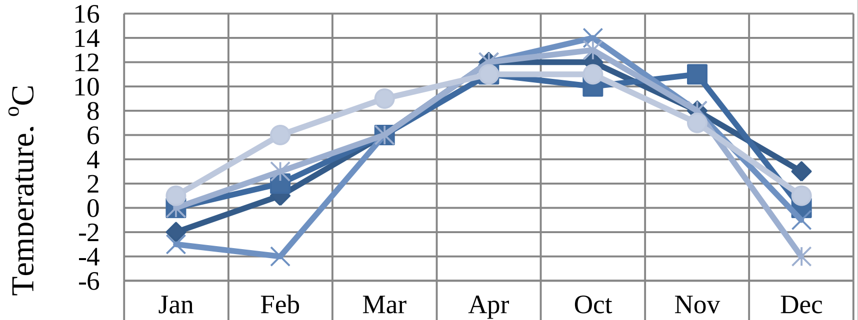


FIGURE 4. Detail of performance of the wall insulation

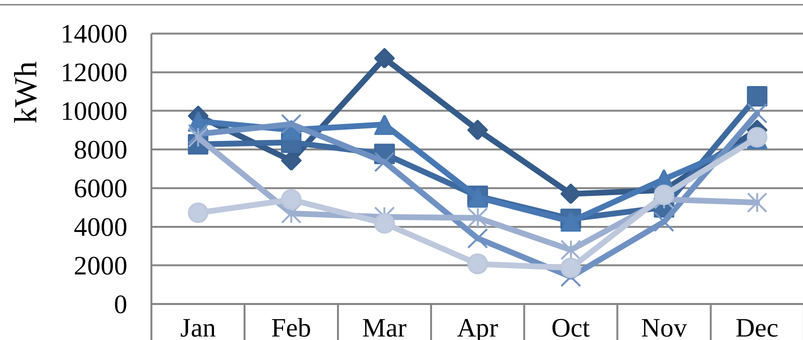
FIGURE 5. E-N Facade after re-habilitation

Figure 6 and Fig. 7 show graphs of average monthly temperatures (Fig. 4) during the heating season of 2009-2014, and electricity consumption (Fig.5) for the same period.Typical of these graphics are sharp jumps consumption in 2009-2010 and significantly smooth increase or decrease in electricity consumption in the period after 2010 i.e. after the refurbishment.



	Jan	Feb	Mar	Apr	Oct	Nov	Dec
2009	-2	1	6	12	12	8	3
2010	0	2	6	11	10	11	0
2012	-3	-4	6	12	14	8	-1
2013	0	3	6	12	13	8	-4
2014	1	6	9	11	11	7	1

FIGURE 6. Average monthly temperatures during season, 2009-2014.



	Jan	Feb	Mar	Apr	Oct	Nov	Dec
2009	9736	7425	12719	9004	5700	5891	9004
2010	8258	8357	7762	5588	4404	4998	10748
2011	9450	9005	9278	5517	4277	6455	8537
2012	8802	9314	7356	3390	1404	4250	9874
2013	8620	4690	4516	4470	2806	5430	5250
2014	4720	5400	4170	2073	1870	5640	8620

FIGURE 7. Consumption of electricity generated for heating, 2009-2014.

Figure 8 and Fig. 9 respectively show graphs of average temperature and average fuel electrical energy for the heating season 2009-2014.There are approximately the same average temperatures for the heating season of 2009, 2010 and 2013 gradually reduce the cost of electricity generated for heating the building, such a reduction is 16% for 2010 compared to 2009 and by 40% in 2013 compared to 2009.

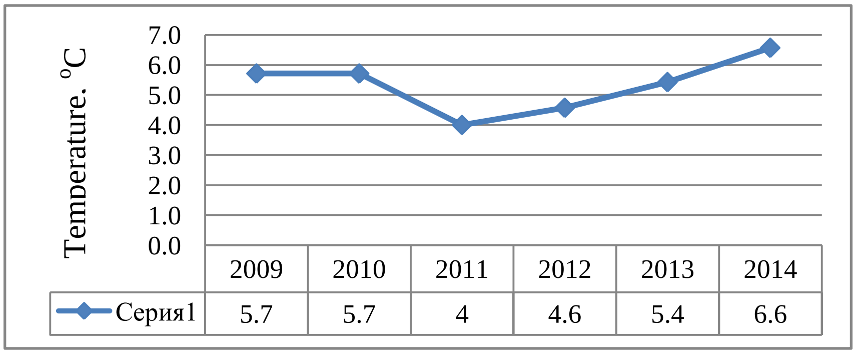


FIGURE 8. Average temperatures for heatingseason.

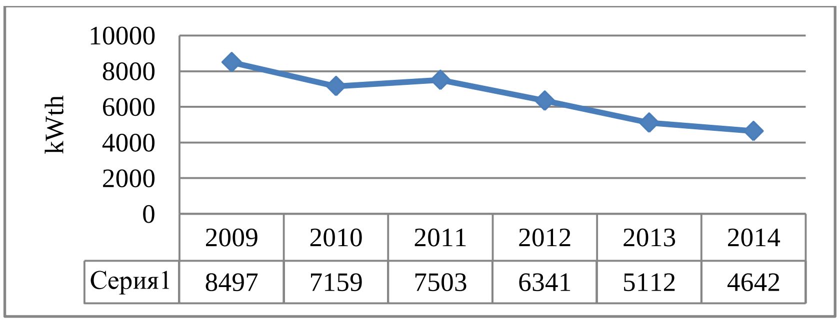


FIGURE 9. Average cost of electricity.

SOME REMARCS

The conclusions are based on observations of an office building with about 24 units for period of seven years: two years ago and renovating five years thereafter. The building has no central heating and each tenant decides how to be heated in the autumn and winter months. By comparing electricity consumption with the average temperature in the months and years results showthat after the renovation costs decrease with each subsequent year, because of few reasons:

- summer temperature of the outer walls of the building is retained longer;
- only tempering premises becomes faster and less power consumed energy because the walls are already playing the role of the battery heat;
- comfortable temperature in the rooms;
- heaters shall be extended for a smooth load schedule of work without spikes;
- savings in power. Energy can help to purchase more expensive, but more efficient heaters with a longer service life and lower power consumption.

If we ignore the price increase of current expenditure for rehabilitation in our case can be compensated for 12 years. But we should be noted that from 2009 to 2015 the price of electricity has changed from 0.112€/kWh to 0.153€/kWh or 36%.

CONCLUSION

Taking into account the reduced energy consumption of electrical charges as a result of the use of energy efficient solutions, such as the remediation costs incurred will be recouped over a period of 10-12 years. The temperature in the premises of 21 - 23^{o}C is achieved and maintained with significantly less heat loss. Energy efficient solutions enable optimum use of space heaters, as well as more flexible temperature control in workplaces both work and overtime. Beyond these financial indicators can point to progress and greater comfort in the workplace, as well as more rational operation of heating

ACKNOWLEDGMENTS

The study has been financially supported by the National Science Fund, Project DFNI E02/10121214, and USEA(VSU) "Lyuben Karavelov", Project 02/2016.

REFERENCES

1. Bettess, P., (1978) *Infinite elements*, IJNME, 1978; 11:54-64.
2. Kazakov, K., (2005) *A model of a FEM type elastodynamic infinite element for Soil-Structure Interaction*, Proceedings of the 4th International Conference on New Trends in the Static and Dynamics of Structures, 20-21 October 2005. Bratislava, Slovakia.
3. Kazakov, K., (2007) *Continuity between Finite and Infinite Elements Along an Artificial Boundary in Soil-Structure Interaction Problems*, Proceedings of the Jubilee Conference in UACG 2007, Sofia, Bulgaria
4. Wolf, J. P., Song, C., (1996) *Finite-element modeling of unbounded media*, England: Wiley, 1996
5. Yan, Ch. B., Kim, D. K., Kim, J. N., (2000) *Analytical frequency-dependent infinite elements for soil-structure interaction analysis in a two-dimensional medium*, Engineering Structures 22 (2000); 258-271.
6. Zhao, Ch., Valliappan, S., (1993) *A Dynamic Infinite Element for Three-dimensional Infinite Domain Wave Problems*, International Journal for Numerical Methods in Engineering, Vol. 36, (1993), 2567-2580
7. Zienkievicz, O. C., Bando, K., Bettess, P., Emson, C., Chiam, T. C., (1985) *Mapped Infinite Elements for Exterior Wave Problems*, IJNME, Vol. 21, (1985), 1229-1251.
8. Directive 2012/27 / EU of the European Union on energy efficiency
9. Energy EfficiencyAct, Prom. SG. 35 of 05.15.2015, effective from 05.15.2015.
10. M. Donn et al., Solution Sets for Net Zero Energy Buildings, Ernst&Sohn, 2015, ISBN 978-3 433-03072-1
11. Kazakov K., A. Yanakieva, *Computational Effectiveness of the Augmented Lagrange Method in the FEM Simulations of Pull-out of Steel bar from Concrete*, Comp. Rendusdel'AcadémieBulgariedesSciences, Tome 65 (2),(2012), 219-224
12. Yanakieva, A. *Stress Determinationin a Cracked Bi-Material Element Usingthe LEFM Energy Approach*. International Journal of MechanicalandProductionEngineering (IJMPE), 3(11), (2015), ISSN:online: 2321-2071/print: 2320-2092