Investigation of Heat Transfer Due To Isothermal Heater in Irregular Porous Cavity: Part I

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Abstract. An investigation is carried out to predict the heat transfer characteristics in an irregular square porous cavity having a small isothermal heater placed at bottom left corner of cavity. The heater is of step shape with vertical right and horizontal top edges which are maintained at an isothermal temperature and right vertical surface of domain is maintained at cold temperature. The top and bottom edges of porous domain are maintained adiabatically. The governing equations are solved iteratively with the help of finite element method. Results are discussed with respect to isothermal and streamline distribution inside the porous domain.

Keywords. Irregular square cavity, Finite element method, Porous medium.

INTRODUCTION

Porous medium refers to a medium that allows the fluid to flow through itself. The flow in porous medium can be studied with respect to various phenomenons such as natural convection, mixed convection, forced convection etc. A thorough understanding of porous medium with respect to various phenomenons is given in literature that includes the books [1-3, 5, 6] and the research articles published in various scientific journals [7-40]. The popular porous geometry that has been studied extensively is that of the square cavity [9, 18, 21, and 23] having the boundary conditions such that any of the walls of cavity being heated and other wall subjected to cooling. The porous domain possesses solid matrix as well as the fluid trapped inside the pores that is responsible to carry the thermal energy due to its movement. The current work is motivated to understand the heat transfer behavior inside the porous region when the irregular porous cavity is heated at a section and cooled at the right vertical wall.

MODEL DEVELOPMENT

Consider an irregular square porous cavity as shown in Fig. 1. The porous medium is confined inside the walls of the irregular cavity. The coordinate system is taken in such a way that x and y points towards the horizontal and vertical directions respectively. The irregular shape of cavity is created due to extraction of step size region in the cavity. The step surfaces i.e., the vertical as well as horizontal surface of step is heated with isothermal temperature T_h whereas the right vertical wall is cooled to temperature T_c as depicted in Fig. 1. The flow in the porous region is governed by Darcy law. The equations that govern the heat as well as fluid flow inside the porous region can be given as.

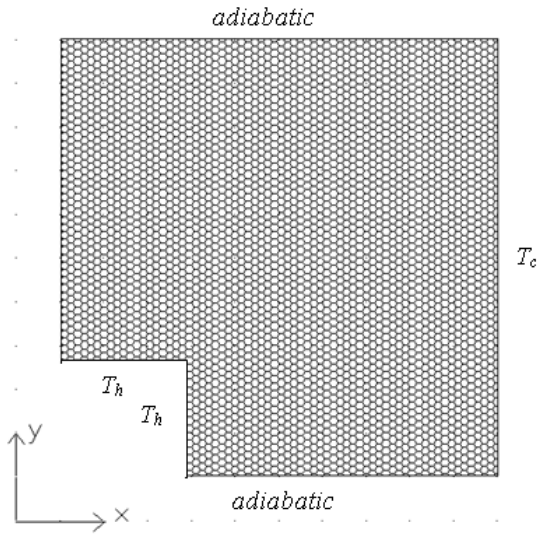

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FIGURE 1. Porous domain

$$\frac{\partial \mathrm{u}}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}=0 \tag{1}$$

$$\frac{\partial \mathrm{v}}{\partial x}-\frac{\partial \mathrm{u}}{\partial y}=\frac{gK\beta}{\nu}\frac{\partial T}{\partial x} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial x} \tag{3}$$

After introducing the Rosseland hypothesis [4] to approximate the radiation q_r parameter,

$$q=-\frac{4n^2\sigma}{3\beta_R}\frac{\partial T^4}{\partial x} \tag{4}$$

The equation (3) becomes:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{1}{\rho C_p}\frac{4n^2\sigma}{3\beta_R}\frac{\partial^2 T^4}{\partial x^2} \tag{5}$$

Taking stream function as

$$u=\frac{\partial \psi}{\partial y} \tag{6a}$$

$$\mathrm{v}=-\frac{\partial \psi}{\partial x} \tag{6b}$$

The boundary conditions are

$$0\le x\le L_s \quad and \quad 0\le y\le L_h \quad u=0 \;\; and \quad v=0 \quad T=T_h \tag{7a}$$

$$x=L \quad u=0 \;\; T=T_c \tag{7b}$$

$$y=0, L_s\le x\le L \quad and \quad y=L \quad v=0 \tag{7c}$$

$$x=0, L_h\le y\le L \quad and \quad x=L \quad u=0 \tag{7d}$$

Where L is the cavity length, L_s is the step length and L_h is step height, at which heat is supplied to porous region.

RESULTS AND DISCUSSION

Equation 2 and equation 3 are the mathematical representation of heat and fluid characteristics that dictates the fluid flow and heat transfer in porous region. These are complex equations which can be solved simultaneously by converting them into alternate form that makes it easy for solution. These equations are converted into dimensionless form and then applied finite element method to convert them into matrix form of equations. The current study focuses on heat transfer due to an isothermal heat source placed in a step shape at the lower left corner of square porous cavity. The step length L_s is varied keeping its height constant at 20% of the cavity length. The contour plot that shows the temperature distribution inside the porous cavity is shown in Fig. 2 and Fig. 3. Fig. 2 shows the effect of step size at which the heat source is supplied and Fig. 3 shows the influence of Rayleigh number. It is observed that the heat transfer at vertical surface of step heater has higher heat transfer rate as compared to the horizontal surface of heater even though both sources have same isothermal temperature. This can be inferred from Fig. 2 that shows the crowded temperature lines near the vertical surface of heater as compared to that of horizontal surface. The larger length of heater pushes the temperature lines towards the cold surface. The maximum value of stream function increased due to increase in the length of the step size as indicated by streamlines of figure 2. The isothermal lines are arranged in a staggered manner with widely spaced lines at low Rayleigh number which shows that the dominant mode of heat transfer is by conduction and convection of thermal energy is minimal at Ra=10. The convection effect increases as the Rayleigh number is increased from 10 to 100 as reflected by higher value of streamlines as well as the oval shaped flow pattern as shown in Fig. 3.

CONCLUSION

Investigation of step size heater placed at bottom left corner of porous cavity is carried out with respect to the length of heater as well as Rayleigh number. It is found that the vertical surface of heater has higher heat transfer rate as compared to its horizontal surface.

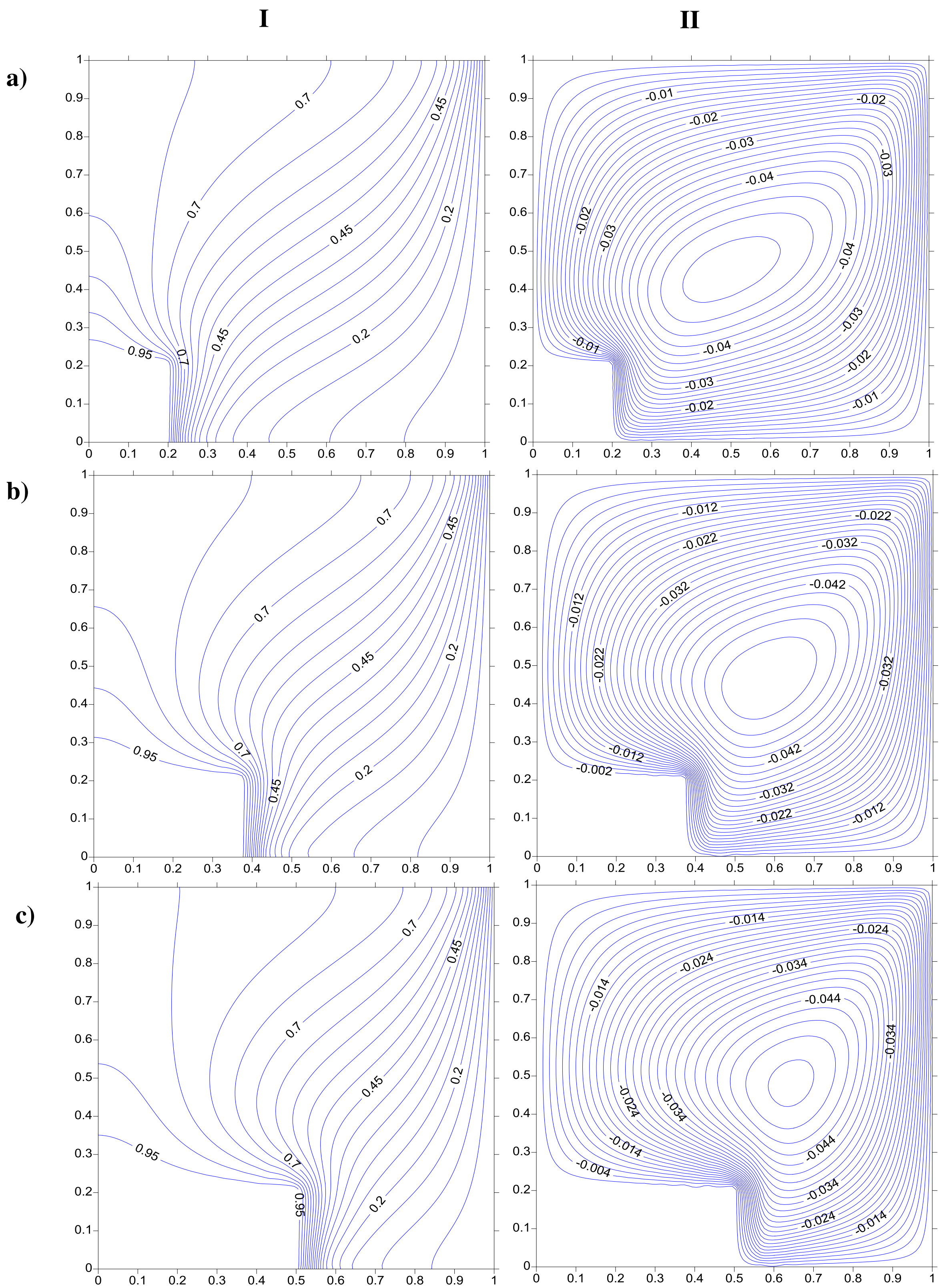

FIGURE 2.I) Isotherms II) Streamlines a) L_s=0.2Lb) L_s=0.375Lc) L_s=0.5L at R_d = 0.1 and Ra = 100

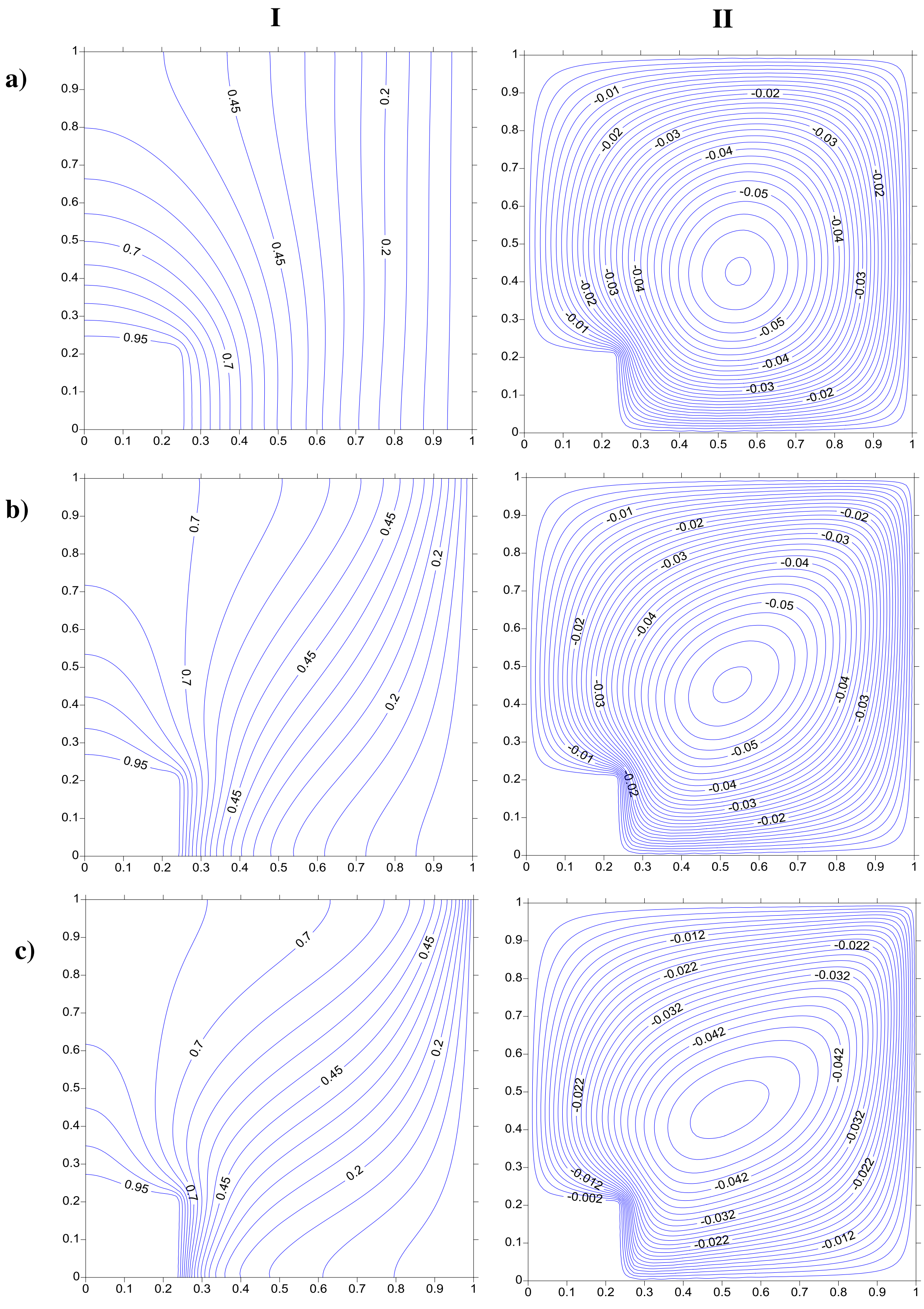

FIGURE 3.I) Isotherms II) Streamlines a) $Ra = 10$ b) $Ra = 50$ c) $Ra = 100$ at $R_d = 0.1$ and $L_s=0.2L$

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