Analytical Study of Temperature Distribution in a Rectangular Porous Fin Considering both Insulated and Convective Tip

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Abstract. The following study is concerned with determination of temperature distribution of porous fins under convective and insulated tip conditions. The authors have made an effort to study the effect of various important parameters involved in the transfer of heat through porous fins as well as the temperature distribution along the fin length subjected to both convective as well as insulated ends. The non-linear equation obtained has been solved by Adomian Decomposition method and validated with a numerical scheme called Finite Difference method by using a central difference scheme and Gauss Siedel Iterative method.

Keywords: Porous fin; ADM; convective tip; insulated tip

INTRODUCTION

Recent innovations in the field of heat transfer have introduced the concept of porous fins. Conceptualized by Kiwan and Nimr [1], the idea of using porous fins is to increase the surface area for convection by improving the effective thermal conductivity. The authors showed in their analysis, an efficient way to save material while transferring same heat than porous fins. Porous fins can be a good replacement of their solid counterparts in cases where weight is a major concern. Modern electronic devices which demand higher heat transfer can be a promising area for fins of this type.

Since its inception, a number of good research works has been devoted to analytical study of porous fins [2-7]**.** Being porous, the fin surface allows fluid to penetrate through the pores that enhances the convective heat transfer. Due to removal of solid material, effective thermal conductivity of the porous fin decreases and thus a sacrifice in heat conduction along the fin length. But the increase in effective surface area may compensate this reduction in heat transfer. Hatami et al [**8]** studied the heat transfer through porous fins made of Si_3N_4 and Al by considering the temperature dependent heat generation. The investigation done by Hatami and Ganji [9] on the efficiency of circular porous fins of different sections revealed that rectangular profiles yield better results compared to convex and triangular shapes.

The current work has been devoted to study the temperature distribution of a rectangular porous fin under convective as well as insulated ends. The differential equation obtained after solving the energy balance equation is highly non linear which was solved by Adomian Decomposition Method (ADM). Careful inspection of the temperature distribution reveals that variation of temperature along the length of a porous fin is always lower than that of its solid counterpart.

METHODOLOGY

A rectangular porous fin is considered in this work. Fig. 1 shows the dimensions of the porous fin of length L, width W and thickness t and it is exposed to a convective environment at constant temperature T_a. The fin is attached to a vertical isothermal wall of uniform temperature T_b.

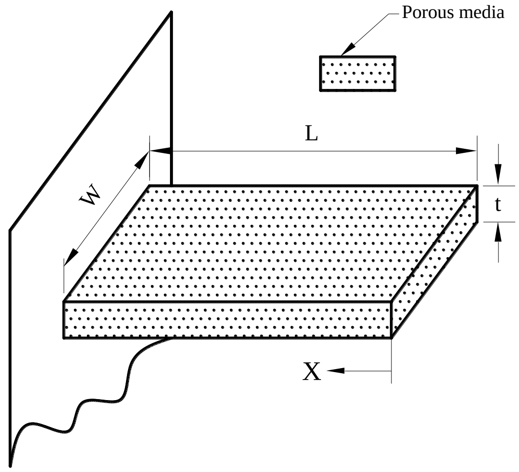


Fig. 1. Schematic diagram of a rectangular straight porous fin

The fin allows fluid to penetrate through it. Darcy model is considered to simulate interactions between the porous medium and fluid. The assumptions considered in this work are:

- Porous medium is homogeneous, isotropic and saturated with a single phase fluid and the physical properties of solid as well as fluid are invariable except density of the fluid that may affect the buoyancy term where Boussinesq approximation is employed.
- Darcy formulation is used to simulate the interaction between the porous medium and fluid and both the mediums are locally thermodynamic equilibrium in the domain.
- There is no contact resistance at the fin base and the temperature variation inside the fin is one-dimensional as the transverse Biot number is very small.
- The temperature inside the fin is function of x only.

By applying energy balance equation in a differential volume of the fin and applying Fourier's law of heat conduction, we get,

$$\frac{d^2T}{dx^2}-\frac{2h(1-\phi)}{k_{eff}t}(T-T_a)-\frac{\dot{m}C_p}{k_{eff}wt\Delta x}(T-T_a)=0 \quad (1)$$

where the mass flow rate,

$$\dot{m}=\rho v w\Delta x \quad (2)$$

Where the fluid velocity is estimated by Darcy's law [2]

$$v=gK\beta\left(T-T_a\right)/\gamma \quad (3)$$

where K is the permeability of the fin material. The effective thermal conductivity of the porous fin can be obtained from following expression:

$$k_{eff}=\phi k_f+(1-\phi)k_s \quad (4)$$

In order to express Eq. (1) in non-dimensional form, the following dimensionless parameters have been introduced as follows:

$$\left(X;\psi;\theta\right)=\left(x/L;t/L;T-T_a/T_b-T_a\right) \quad (5a)$$

$$\Theta=k_{eff}/k_f=\phi+k_R-\phi k_R \quad (5b)$$

$$\left(S_1;S_2\right)=\left(\frac{Ra\,Da}{\psi^2\Theta};\frac{2Nu\left(1-\phi\right)}{\psi^2\Theta}\right) \quad (5c)$$

$$\left(Ra;Da;Nu\right)=\left(\frac{\rho C_p g\beta\left(T_b-T_a\right)t^3}{\gamma k_f};\frac{K}{t^2};\frac{ht}{k_f}\right) \quad (5d)$$

Using Eqs. (2)-(5), Eq. (1) can be expressed as

$$\frac{d^2\theta}{dX^2}-S_1\theta^2-S_2\theta=0 \quad (6)$$

As stated earlier, both the fin tip conditions (insulated and convective) are considered. Thus to determine temperature distribution, boundary conditions of Eq. (6) can be written as follows:

(i) $\theta=1$ at $X=1$ for both insulated and convective tip (7a)

$$\text{(ii)}\ \left.\frac{d\theta}{dX}\right|_{X=0}=\begin{cases}0 & \text{for insulated tip}\\ H=\left(Nu/\Theta\psi\right)\theta\left(0\right) & \text{for convective tip}\end{cases} \quad (7b)$$

The equation obtained is non-linear in kind which cannot be solved using traditional techniques. Thus an analytical technique called ADM is used to solve the equation.

Now by applying ADM [8], the temperature distribution can be obtained as follows:

A. *Considering convective tip*

$$\theta=z+F_1\frac{x^2}{2!}+F_2\frac{x^4}{4!}+F_3\frac{x^6}{6!}+.... \quad (8)$$

$$F_1=(S_1z^2+S_2z);F_2=(2S_1Fz+S_2F_1);F_3=6S_1F_1^2 \quad (9)$$

B. *Considering insulated tip*

$$\theta=z+P_1\frac{x^2}{2!}+Q_1\frac{x^3}{2!}+(R_1+P_2)\frac{x^4}{4!}+(R_2+Q_2)\frac{x^5}{5!}+... \quad (10)$$

$$\begin{aligned}&P_1=S_1z^2+S_2z;\ \mathrm{Q}_1=2Az^2S_1+S_2Az\ ;\mathrm{R}_1=2A^2z^2S_1;\\&P_2=2P_1S_1z+P_1S_2;\ \mathrm{Q}_2=2Q_1zS_1+6P_1AzS_1+Q_1S_2\ ;\\&\mathrm{R}_2=2R_1S_1z+8AzQ_1S_1+R_1S_2\end{aligned} \quad (11)$$

The temperature distribution for both the cases described in aforementioned section is a function of unknown tip temperature z that can be determined by applying boundary condition (7a) and solving the corresponding transcendental algebraic equations using Newton-Raphson iterative method and after satisfying the necessary convergence criteria, final tip temperature is obtained.

RESULTS AND DISCUSSION

The results obtained from the analytical study are compared with a numerical scheme called Finite Difference method by using a central difference scheme and Gauss Siedel Iterative method. The analytical results seem to coincide with that of numerical results which validates the present work. A comparative analysis between solid and porous fin has been shown in Fig. 2.

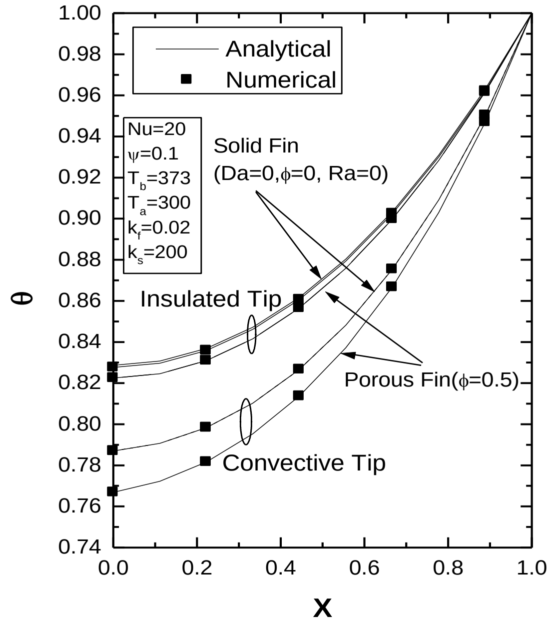
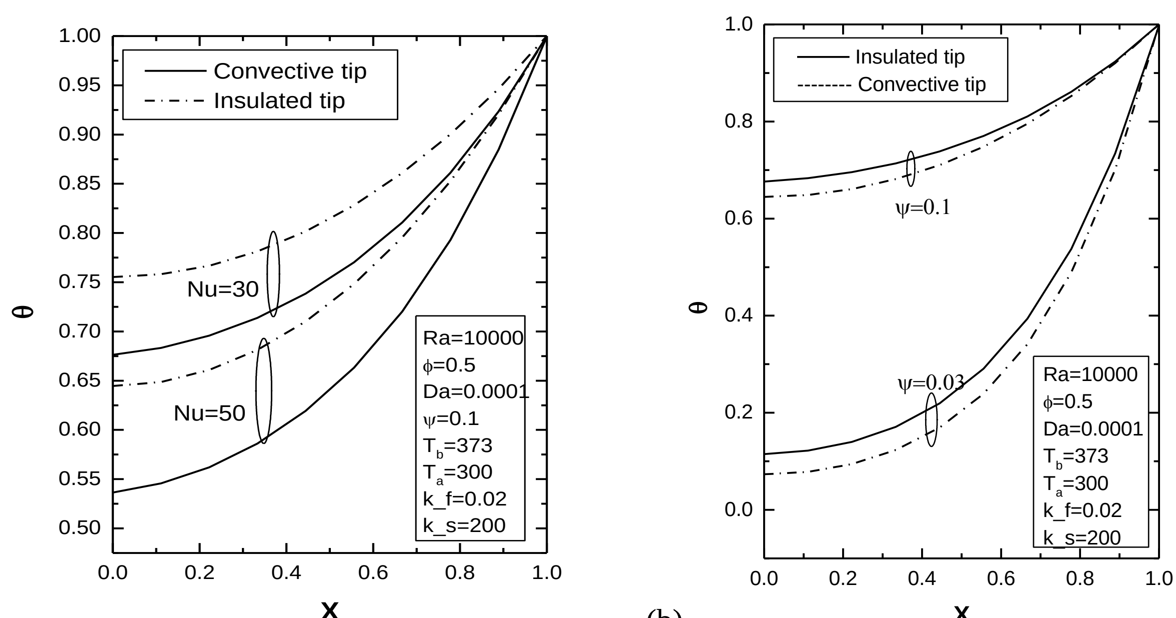
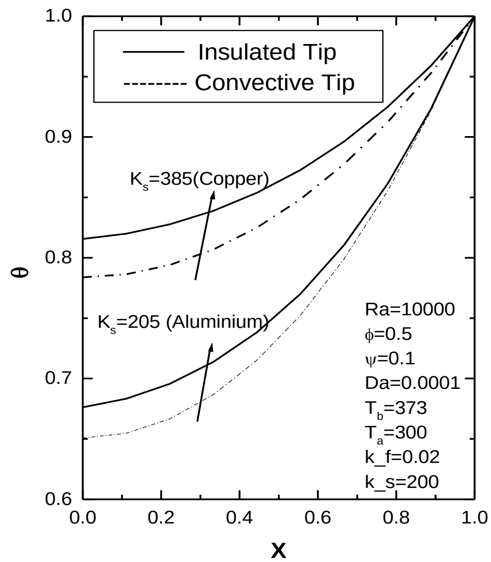


Fig.2. **C**omparisons of temperature distribution between analytical and numerical results of both solid and porous fin

For both insulated as well as convective tip conditions, the temperature distribution in solid fins shows an increasing trend compared to the porous one. Convection through pores facilitates better heat transfer rate in case of porous fins which is why we get this trend.



(a) (b)



(c)

Fig. 3. Temperature distribution of a porous fin (a) variation obtained by varying *Nu*; (b) effect of ψ; (c) A comparative analysis between Al and Cu fin

In Fig. 3(a), the role played by Nusselt number (*Nu*) is studied. As *Nu* increases, the convective heat transfer dominates and transfer more heat from the fin body. The temperature profile as shown in the figure is in agreement with the above logic. For both the cases the temperature profile in case of fin with insulated tip is higher than that obtained in case of fin with convective tip. ψ is an important parameter in the design of porous fins. With decreasing thickness, the temperature along the fin drops drastically as seen in Fig. 3(b). This is quite obvious as with less thickness the surface area for convection as well as conduction along the body drops. Copper being more conductive than Aluminium shows higher temperature distribution along the fin length as depicted in Fig. 3(c). The thermal conductivity of fins made of copper allows them to conduct and carry away more heat as compared to Aluminium. However Aluminium has its own set of benefits, such as light weight and less expensive, which makes them a better choice in most of the applications.

IV. CONCLUSIONS

The above study introduces a simple way to determine the temperature distribution of porous fin under convective medium. The fin considered for the current analysis is of rectangular shape and dissipates heat through natural convection. The governing differential equation obtained is highly non-linear which is solved using Adomian Decomposition Method. The study takes in to account a number of parameters which plays a major role in heat convection through porous fins. Following key conclusions can be stated from the analysis:

- ψ plays a major role in heat transfer through fins. Thus care should be taken to select this parameter judicially.
- At higher Nusselt number the temperature along the fin drops. The reason for this trend can be credited to the increased convection through pores.
- Convection through pores enhances the transfer of heat, which is why porous fins are more efficient in heat transfer compared to their solid counterparts.

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