Hyperchaos generated from 3D chaotic systems using PI Controller

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Abstract. Hyperchaotic systems have chaotic behavior with at least two positive Lyapunov exponents and the minimum required system dimension is four. In this paper, a 3D chaotic system is converted into hyperchaotic system using PI controller in the feedback path. Integral controller is responsible for increase in order of the system. The hyperchaotic nature is verified by the existence of two positive Lyapunov exponents and using bifurcation diagrams. The system is hyperchaotic in several different regions of the parameters. The result shows that this method can not only enhance or suppress chaotic behavior, but also induces chaos in non-chaotic parameter ranges. The proposed method is applied to a secure communication system by means of encryption and decryption of a message using hyperchaotic system.

INTRODUCTION

A large number of hyperchaotic systems were developed [2–9] after the first report on hyperchaos principle by Rossler in 1979 [1]. Hyperchaotic attractors received very much interest in researchers due the wide scope in communication and control technologies. Designing a hyperchaotic system using feedback controllers in originally chaotic but non-hyperchaotic system, is theoretically enchanting and yet technically quite challenging mission.

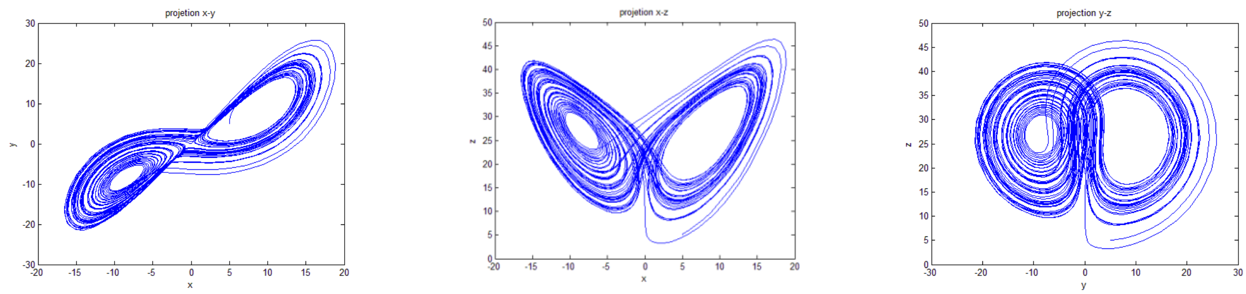
In this paper, a hyperchaotic system is generated with the use of a PI controller in feedback, thereby exploring the alterations of a 3D chaotic system. A new system based on the 3D chaotic system of Lorenz is introduced in this article by adding a new state variable. It is shown that the proposed system display the characteristic of hyperchaos and chaos. In addition, the attractors of the system have very strange shapes which are distinctly different from those of the traditional chaotic attractor. All the simulations are done using MATLAB.

DESIGN OF HYPERCHAOTIC SYSTEM

Lorenz system [11] is described as

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - y - xz \\ \dot{z} &= xy - bz \end{aligned} \qquad \text{…….(1)}$$

When a = 10, b = 8/3, c = 28, Lorenz system exhibits a chaotic behavior.



(a) (b) (c)

Fig.1 Projections of Lorenz system for a=10,b=8/3 and c=28 (a) x-y projection, (b) x-z projection, (c) y-z projection

Some important basic features of this system are:

1. It is autonomous, which means that time does not explicitly appear on the right hand side.
2. The equations involve only first order time derivatives, so the evolution depends only on the values of x,y, and z at the time.

3. The presence of terms of xz and xy in the second and third equations make the system is non-linear.
4. The system is dissipative when the following inequality holds:

$$\nabla f = \frac{d\dot{x}}{dx} + \frac{d\dot{y}}{dy} + \frac{d\dot{z}}{dz} = -a - 1 - b < 0$$

5. Since parameters a and b, denoting the physical characteristics of the air flow, are positive, the inequality always holds and, thus, solutions are bounded.
6. The system is symmetric, with respect to the z axis, which means it is invariant for the coordinate transformation:
   (x,y,z)→(−x,−y,z)

Proposed Method

A PI controller *w* is added to the equation (1) of above system.

$$w = k_p\, x + k_i \int x\, dx$$

Where k_p and k_i are the proportional and integral parameters.
The major requirements of hyperchaotic systems are as follows:

1. The system should have dissipative nature
2. The minimum dimension of the phase space is four.
3. The number of terms in the equations giving rise to instability is at least two, of which one should be a nonlinear function.

The resultant system equations are

$$\begin{aligned} \dot{x} &= a(y - x) + w \\ \dot{y} &= cx - y - xz \qquad \text{…….(2)} \\ \dot{z} &= xy - bz \\ \dot{w} &= k_p \mathrm{a}(\mathrm{y} - \mathrm{x}) + k_i x \end{aligned}$$

In system (2), k_p and k_i are the control parameters. The system undergoes a change from hyperchaos to limit cycle when the parameter varies.

Behavior of the controlled system

Dissipative Nature

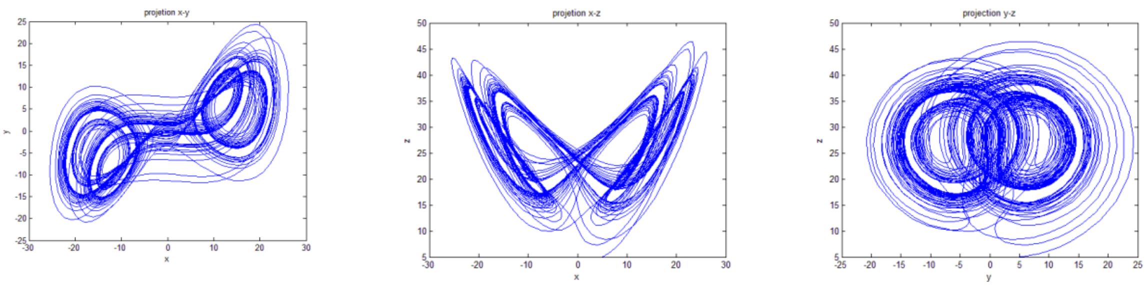
With a = 10, b = 8/3, c = 28, then

$$\nabla V = \frac{d\dot{x}}{dx} + \frac{d\dot{y}}{dy} + \frac{d\dot{z}}{dz} + \frac{d\dot{w}}{dw} = -10 - 1 - \frac{8}{3} = -13.66$$

And the system is dissipative.

Lyapunov Exponent

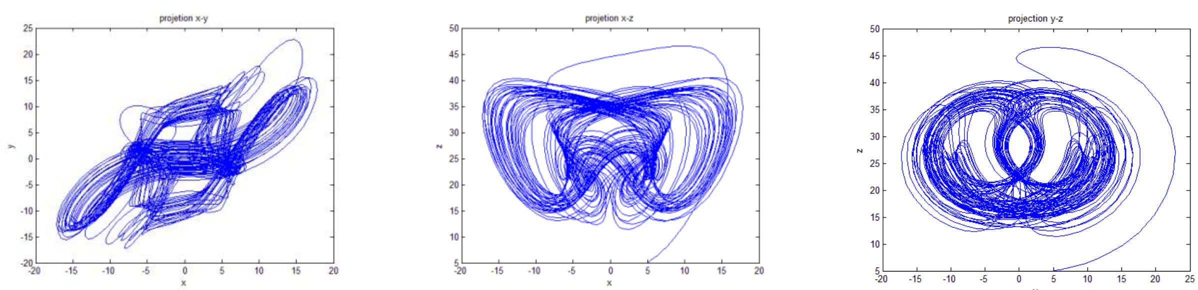
With k_i = 1.3, the Lyapunov exponents obtained are: 1 = 0.01000, 2 = 0.421905, 3 =-0.326781, 4 = −13.385272. Since more than two Lyapunov exponents are positive, the system is hyperchaotic [13]. The results obtained by simulation are shown in Fig. 2. The projection of attractor in three different plans are shown in figures a, b & c.



(a) (b) (c)

Fig.2. Projections of hyperchaotic system with k_p=-3.6, k_i=4.8 : (a) x-y projection, (b) x-z projection, (c) y-z projection

The projections of proposed system with parameter k_p=-4.6, k_i=5.2 is shown in Fig.3



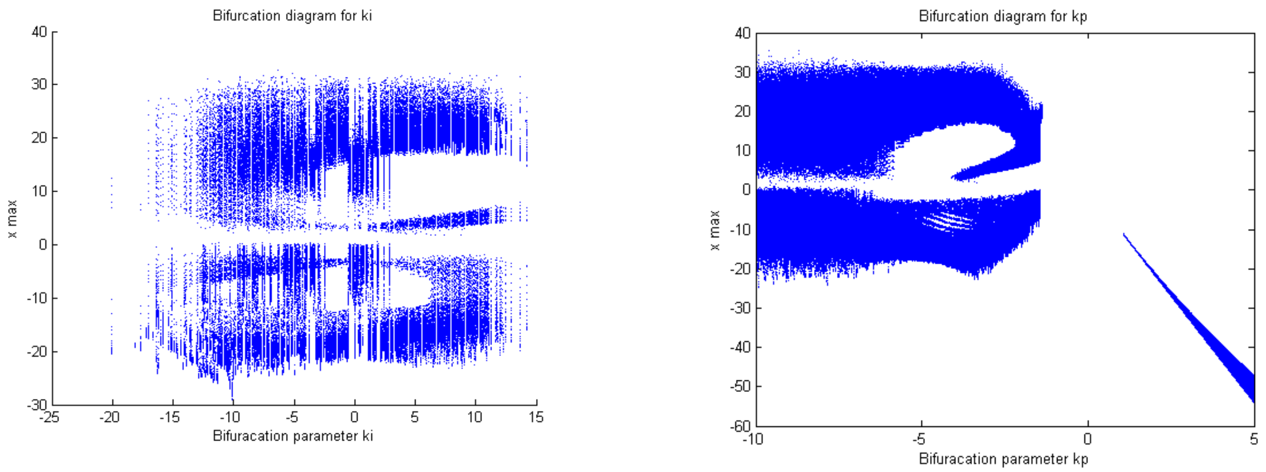
(a) (b) (c)

Fig.3 Projections of hyperchaotic system with k_p=-4.6, k_i=5.2 : (a) x-y projection, (b) x-z projection, (c) y-z projection

Bifurcation diagrams

The most useful graphical representation of the sequence of bifurcations that take place in the system when the control parameter changes is bifurcation diagram. The bifurcation diagram obtained with varying parameter values of k_p and k_i is shown in Fig 4. In the first plot, k_p is fixed and k_i is varied and in the second k_i is fixed and k_p is varied.

For k_p = -3.5, a = 10, b = 10/3, c = 28 and for varying k_i, the bifurcation diagram shows at different values of k_i the system can have hyperchaotic behavior. The diagram is shown in Fig.4.1



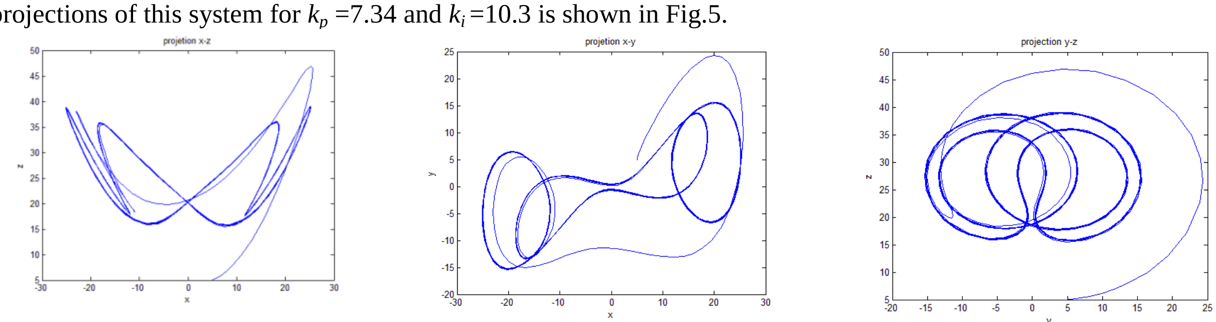
(a) (b)

Fig.4 Bifurcation diagram : (a) with k_p =-3.5,a=10,b=8/3, c=28 and varying k_i (b) with k_i =6.5,a=10,b=8/3, c=28 varying k_p

For k_i=6.5,a=10,b=8/3 , c=28 and for varying k_p the bifurcation diagram shows at different values of k_p the system can have chaotic and hyperchaotic behavior.

Suppression of chaos

The chaotic behavior of the system can be suppressed by changing any of the parameters k_p or k_i. It is easy to understand from the bifurcation diagram shown in Fig.4 that for k_p >0 the system behave non-chaotically. The projections of this system for k_p =7.34 and k_i =10.3 is shown in Fig.5.

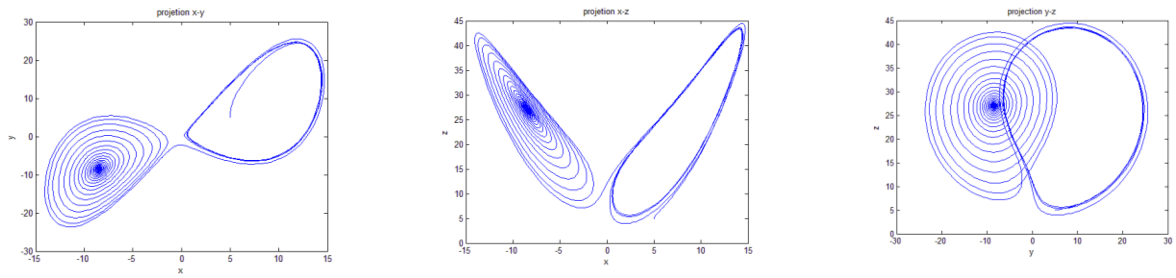


(a) (b) (c)

Fig.5. Behavior of the system with k_p =7.34, k_i =10.3 : (a) x-y projection, (b) x-z projection, (c) y-z projection

Chaotification in the nonchaotic parameter range

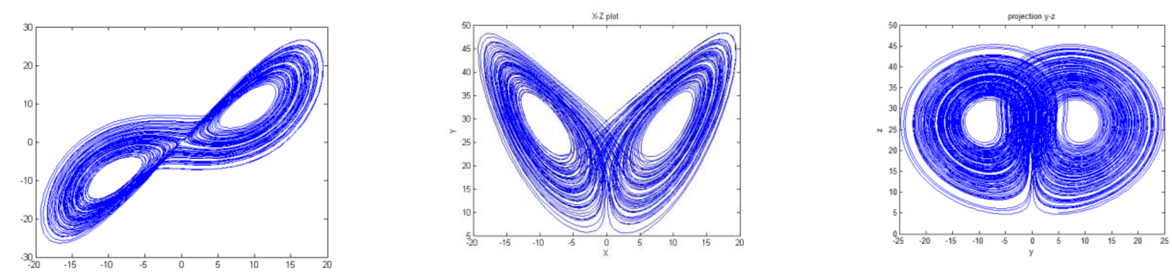
The Lorenz system is non-chaotic for a wide range of parameter values. For a typical case with parameter values a=4, b=8/3 & c=28, the projections are shown in Fig.6. The simulation results clearly establish that the system is non-chaotic.



(a) (b) (c)

Fig.6. Non-chaotic behavior for the parameter a=4, b=8/3 and c=28 : (a) x-y projection, (b) x-z projection, (c) y-z projection

It is possible to induce chaotic behavior in the system by using the PI controller. It can be shown that the system can be made chaotic for wide range of values: k_p <-2.5 and k_i >0. The projections of the system with k_p =-2.7 & k_i=4.9 is shown in Fig. 8

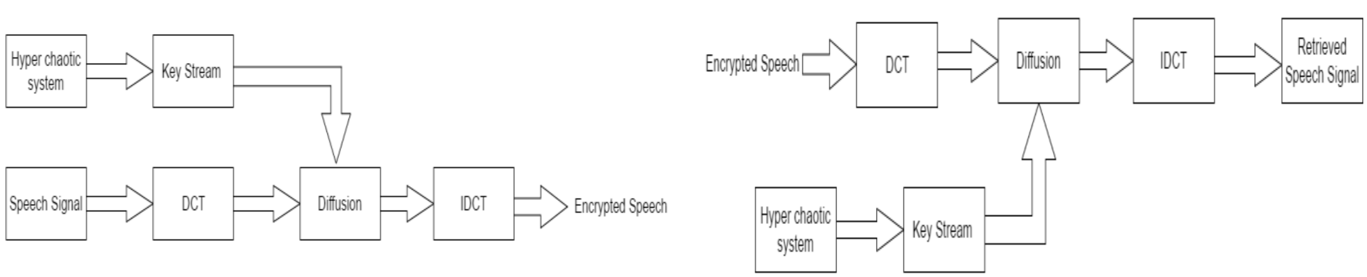


(a) (b) (c)

Fig.8. Chaotic behavior with PI controller : (a) x-y projection, (b) x-z projection, (c) y-z projection

APPLICATION

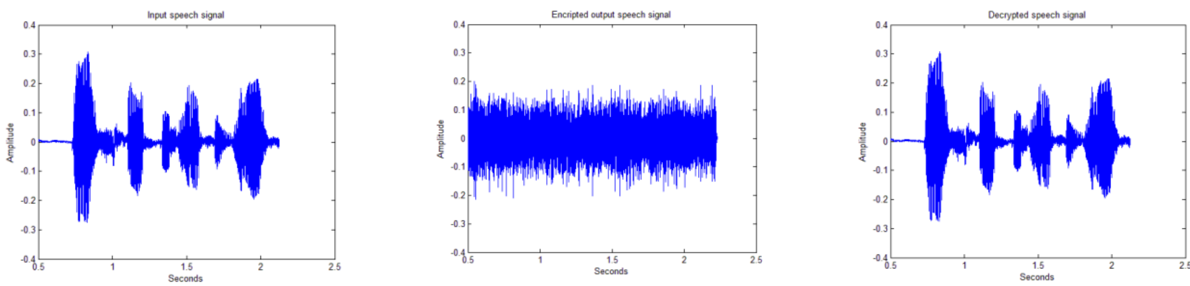
The hyper chaotic Lorenz system can be used for the secure communication of speech signals. The block diagram for encryption and decryption [14] are shown in Fig.9



(a) (b)

Fig.9 Block diagram of Secure Communication : (a) Encryption of speech Signal using Hyper chaotic system (b) Decryption of speech Signal using Hyper chaotic system

Initial conditions and system parameters are the key function which is given as the input to the encryption system. Diffusion helps to fill the silent part of the speech by noisy samples, which is the integral part of the encryption process. At the sender part, key stream has been generated using the proposed hyperchaotic system. Before the encryption process the speech signal is converted to frequency domain by discrete cosine transform. Speech scrambles are diffused by XOR operation. After the diffusion process transform domain encrypted speech signal is converted in to time domain by inverse discrete cosine transform. Encrypted voice is transmitted through channel after modulation. Retrieval of the original speech signal is carried out at receiver section. The encrypted voice is converted to transform domain by applying DCT prior to the diffusion process. Apply inverse discrete cosine transform to restore the original speech signal. Decryption process has the same structure as the encryption process. The Receiver should generate the same key stream to restore the original signal. Therefore the sender and receiver should be provided with same keys (initial values and control parameters).The simulation results shown in fig 10. Encryption and decryption has been successfully carried out.



(a) (b) (c)

Fig.10 Encryption and Decryption using Hyperchaotic system : (a)Speech input (b)Encrypted Data (c)Decrypted Data

CONCLUSION

Detailed study of 3D Lorenz system with PI controller in the feedback is carried out. By suitable varying the control parameters of PI controller, it was possible to make the system hyper-chaotic and non-chaotic. It is also possible to induce chaos in the system with in non-chaotic parameter range. A typical application of the hyper-chaotic system in Speech encryption and decryption was also studied.

REFERENCES

1. O.E. Rosler, An equation for hyperchaos, Phys. Lett. A 71 (1979) 155–157.
2. T. Kapitaniak, L.O. Chua, Hyperchaotic attractor of unidirectionally coupled Chua’s circuit, Int. J. Bifur. Chaos 4 (1994) 477–482.
3. K. Thamilmaran, M. Lakshmanan, A. Venkatesan, A hyperchaos in a modified canonical Chua’s circuit, Int. J. Bifur. Chaos 14 (2004) 221–243.
4. Y. Li, G. Chen, W.K.S. Tang, Controlling a unified chaotic system to hyperchaotic, IEEE Trans. Circuits Syst.-II 52 (2005) 204–207.
5. R. Barboza, dynamics of a hyperchaotic lorenz system, Int. J. Bifur. Chaos 17 (2007) 4285–4294.
6. X. Wang, M. Wang, A hyperchaos generated from the Lorenz system, Physica A 387 (2008) 3751–3758.
7. Q. Yang, K. Zhang, G. Chen, Hyperchaotic attractors from a linearly controlled Lorenz system, Nonlinear Anal. RWA 10 (2009) 1601–1617.
8. Q. Yang, Y. Liu, A hyperchaotic system from a chaotic system with one saddle and two stable node-foci, J. Math. Anal. Appl. 360 (2009) 293–306.

9. Y. Liu, Q. Yang, G. Pang, A hyperchaotic system from the Rabinovich system, J. Comput. Appl. Math. 234 (2010) 101–113.
10. Y. Li, W.K.S. Tang, G. Chen, Hyperchaos evolved from the generalized Lorenz equation, Int. J. Circuit Theory Appl. 33 (2005)
11. E.N. Lorenz, Deterministic nonperiodic flow, Journal of Atmospheric Sciences 20 (1963) 130–141.
12. Thabet Hajer, Seddik Hassene, Generating a Hyper-Chaotic System from 3D Chaotic Behaivor, International Conference on Advanced Technologies for Signal and Image Processing - ATSIP'2016,(2016) 46-51.
13. Farsana F J,Dr K Gopakumar, Speech Encryption Based on Four-Dimensional Hyperchaotic System, International Conference on Data Mining and Advanced Computing (SAPIENCE), (2016).